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Rapid Determination of Geoacoustic Properties of the Sea Floor by Simulated Annealing: Initial Report

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The Naval Research Laboratory (NRL) developed and tested a simulated annealing (SA) inversion method for geoacoustic data. Significant improvements in the classical Metropolis SA algorithm improved the speed and accuracy of the SA method. This method solves for compressional and shear wave velocities, layer thicknesses, density, and attenuation when the data are sensitive to these properties. NRL demonstrated the effectiveness of the SA method with a realistic geological case that is a nonlinear problem with several traps that foil other inversion methods.

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INTRODUCTION

Current world conditions require that the Navy has the ability to rapidly make geoacoustic surveys in areas that were previously of little interest. Past and current survey methods are cumbersome and slow. This report addressed this issue by developing a rapid method of solving for the environmental properties within a survey area. This method can be applied to a variety of geoacoustic data such as multichannel marine seismic data but, in particular, we wish to acquire geoacoustic data with an array of sonobuoys and SUS charges that are fairly randomly arranged (i.e., no rigorous geometric constraints required). Then, using these data, we solve for the subbottom geoacoustic parameters such as compressional wave velocity (V_p) illustrated in Fig. 1.

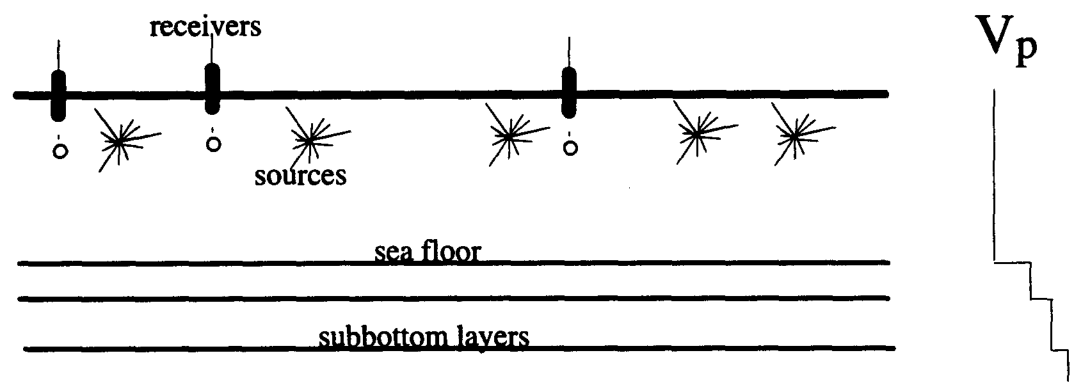

Fig. 1—Cross section of subbottom profiling experiment with floating receivers and multiple pulse sources. V_p values are plotted at the right.

To find a solution by our method we need to search over a large model space (Fig. 2).

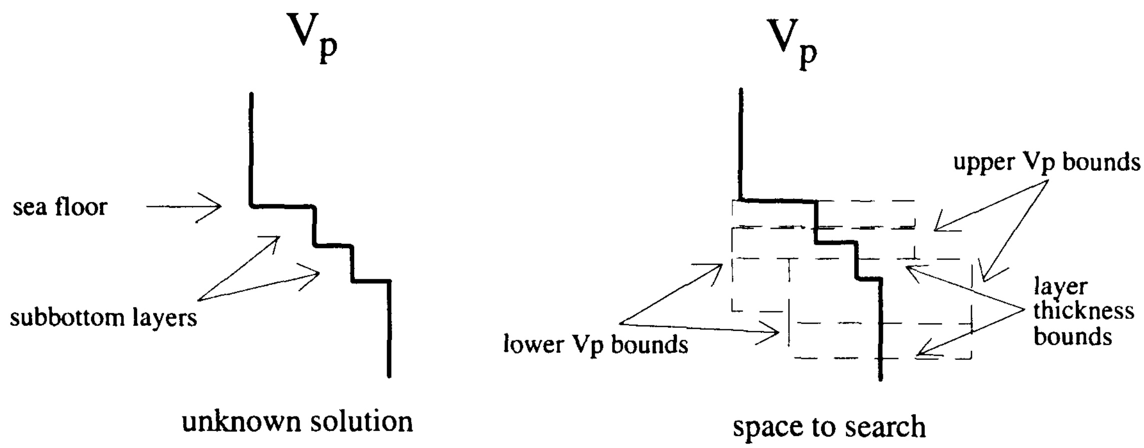

Fig. 2—Illustration of the search over model space to find the true values of V_p. The left figure illustrates the true but unknown values of V_p, and the right figure illustrates the lower and upper bounds of the V_p and layer thickness values for the search.

Using a relatively simple 6-layer model for continental shelf sediments with resolutions possible with high-frequency multichannel geoacoustic systems (1 to 10 m for layer thickness and 10 to 100 m/s for V_p) there are $1.1 * 10^{18}$ solutions in this model space. Sampling one solution every 3.25 s (the actual rate on a Sun SPARC 10 desktop workstation), it would take 34,000,000,000 years to do a complete search. Using Simulated Annealing (SA), a directed search method, we found a solution with maximum errors of less than 5% and average errors of 1% to 2% for both layer thicknesses and sound velocities in 2 h and 25 min (Fig. 3). If this inversion was run on a massively parallel computer, then we would expect run times of a few minutes.

SA is a nonlinear inversion method (see Appendix) that searches model space in a pseudo random fashion that is directed by a statistical selection criterion (Kirkpatrick et al. 1983). The search over model space is done by altering one variable in the current model at a time, creating a new model, and then calculating synthetic data with a forward model routine. Synthetic data are compared to the field data using an objective function. A variety of objective functions can be used, such as cross correlation or residual (Sen and Stoffa 1991). If the new model produces a lower objective function, then this new model is chosen as the current model and the process is repeated. If the new model produces a higher objective function, then it is occasionally chosen as the current model by a method that resembles the statistical energy distribution of atoms in a solid. At a high "temperature," the chance of choosing the solution with a higher cost function is much higher than at a low "temperature." This process is begun at a high temperature and repeated hundreds or thousands of times as the temperature is slowly reduced in a process that is mathematically analogous to annealing a solid object. If the temperature is reduced slowly enough over the proper range, then the solution, or object, will "anneal" into a single crystal, which is the lowest energy state possible, and the synthetic data calculated will be the best possible fit to the field data (van Laarhoven and Aarts 1987). See the appendix for more details about SA.

Current Status of the Inversion Method

The working SA algorithm is based on the classical Metropolis algorithm (Metropolis et al. 1953) (see Appendix) but has significant differences in all major steps. Two values used throughout the algorithm are the objective function and the temperature. The objective function is used to measure the goodness-of-fit between the synthetic data calculated from the test model $\underline{\boldsymbol{m}}$ and the field data (with a small value being a better fit than a large value), and the temperature is a parameter used in selecting or rejecting a bad fit and to adjust the size of the model parameter search window. We can use any of five different objective functions: the zero lag cross correlation, a modified cross correlation that retains amplitude information, the residual (sum of the differences), the residual squared, and the square root of the residual.

We use a temperature dependent search distribution and a layer dependent temperature. A temperature dependent search distribution gives a much finer search pattern near the minimum. The layer dependent temperature fits the top layers first and is more efficient for a model with many layers as it results in a linear increase in the run time relative to the number of layers rather than a geometric or exponential increase in time.

We tested the SA algorithm by inverting synthetic data. The synthetic data are simplified and noise-free examples of what is expected from real geological sites. The synthetic data simulate field data collected by sonobuoys using SUS charge sources as well as field data from high-resolution multichannel systems such as the Deep Towed Acoustics-Geophysics System (Gettrust and Ross 1990). We can invert seismic or geoacoustic data in the tau-p domain, the t-x domain (time and offset, the most direct observation), or the ω-x domain (frequency and offset) with any of five different objective functions. Geoacoustic data are normally collected in the t-x domain and transformed to the tau-p or the ω-x domains if desired. However, a reliable tau-p transformation requires more offsets (i.e., sonobuoy-SUS pairs) than necessary for the inversion done in the t-x domain.

Figures 3 and 4 are examples of simulated field data and inversion solutions. The example shown in Fig. 3 uses the expected geoacoustic profile for a thick sedimentary basin such as a continental shelf, specifically the Nile Fan in the Mediterranean Sea. These synthetic data are shown in the tau-p domain (intercept time and ray parameter, a plane-wave decomposition of the wave field), which makes for a faster inversion calculation. Figure 3a shows the synthetic "data" in tau-p space. Figure 3b shows the residual or the difference between the "data" and synthetics calculated from the solution or fit found by the inversion calculation. The low amplitudes here indicate that the solution model is close to the true model (the fit is good). Figure 3c shows the cooling curve or the progress of the fit as the calculation proceeds from a high temperature and large objective function on the right side to a low temperature and small objective function on the left side. The objective function is best defined so that a perfect fit gives a value of 0 and a completely random guess gives an average value of about 1. Figure 3d compares the model used to generate the "data" with the solution found by inversion. The solid line profile was used to calculate the "data" and the dashed line profile is the solution from the 19th run of our SA inversion method. We solved for layer thickness and V_p. The closeness of the two profiles demonstrates that this solution is good.

Figure 4 shows an interesting geological situation found with a multichannel seismic marine survey; that of one or two thin layers having high V_s over otherwise normal sediment. Figure 4a shows the synthetic "data" in t-x space. The offsets here are regularly spaced but need not be. Irregular spacing as from sonobuoys and SUS charges will work well in this domain. Figure 4c shows the cooling curve that shows a decreasing objective function at the lowest temperature suggesting that an even better fit could easily be achieved. However, the fit is very close and the residual is very small (Fig. 4b). Figure 4d compares the model used to generate the "data" with the solution found by inversion. The resolution and frequency are much higher than in Fig. 3 so that the thin layers can be resolved. The only variable searched over in the inversion was the shear velocity (V_s) giving only three variables. This inversion was done without layer stripping but with a temperature dependent search window. The inversion done in the t-x domain is very slow even though the search was only over three variables.

The advantage that SA (or other nonlinear inversion methods) has over linear methods is demonstrated in Fig. 5, which is a gray tone contour plot of the objective function for V_s of the first and second layers for the model shown in Fig. 4. The objective function used here is the negative of the cross correlation so that a perfect fit has a value of 0 while random models give an average value of 1. The black area is where the objective function has a value near 0 and the true V_s values are 500 m/s for layer 1 and 650 m/s for layer 2. The gray areas below it are local minima. The highest contour level on this plot is at 0.1, which is a fairly good fit but this covers much of the model space. The complex shape and multiple minima are a challenge for the inversion method. A linear inversion method will easily fall into one of the local minima and the user may be satisfied with an objective function of 0.06 to 0.08, but will end up with the wrong solution. Our SA method easily found the global minimum every time, even when the search was done very rapidly. Other objective functions calculated for the same model give similar plots with multiple local minima. Any model with thin layers will be nonlinear and have local minima in their objective function meaning that a nonlinear search method such as our SA algorithm is needed to automatically find the optimum environmental model.

Plans for the Future

Our SA methods work well with synthetic data in the tau-p, t-x, or ω-x domain. While the tau-p domain inversion is much faster than the t-x domain inversion, real data are normally collected in the time-offset (t-x) domain and then transformed to tau-p or another data domain for the most efficient analysis. A reliable transformation depends on adequate coverage of offsets, which is an important experimental design parameter. Realistic experiment designs never have adequate offset coverage for a good tau-p transformation making the subsequent inversion in that domain unreliable. Inversion in the frequency-offset (ω-x) domain avoids this offset coverage problem but is currently no faster than the t-x domain inversion. We will develop a much more efficient ω-x forward model during the current year.

SA inversion produces a single "best" solution which, if done properly, will be very close to the "true" solution. It will not tell how sensitive the data are or the solution is to the different parts of the physical model. For example, typical sea-floor data will be very sensitive to compressional velocity differences in thick layers but not in thin layers. This information is not included in the solution, and the "best" solution may be substantially different from the "true" solution for the model parts for which the data are insensitive. The "Heat Bath" method is similar to SA but produces a probability density function that gives a "best solution" at its peak but also shows how reliable the solution is by its width (Basu and Frazer 1990). During the current year, we will incorporate the Heat Bath method as an option to the inversion in all of the data domains mentioned above.

Many improvements will be made to the algorithm, such as cleaning up the inversion code, improving the data windows, making the model search window a function of the value of the objective function and the temperature, using a variable dependent temperature, using a better normalization of the objective functions, making layer acoustic travel times independent of the layer thickness, using improved cooling schedules, and investigating the sensitivity to noise. We will investigate these and other ideas this year and incorporate all of those that prove worthwhile into our method.

This year we plan to apply this method on field data such as those collected during the Critical Sea Test-8 experiment last spring and during upcoming cruises gathering data that will be appropriate for our analysis methods.

Summary

We developed and tested an SA algorithm for inverting synthetic geoacoustic data from a marine environment. We use several significant improvements in the classical Metropolis SA algorithm that improve the speed and accuracy of the method, such as using variable dependent temperatures and a temperature dependent model search window. This method works with three data domains that are either optimal for data collection or forward model calculation, but not both. We demonstrated the effectiveness of the SA method with a realistic but unexpected geological case that is a nonlinear problem with several traps that foil other inversion methods.

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Dr. L. Neil Frazer of the University of Hawaii and Dr. Mrinal Sen of the University of Texas made major contributions through their discussions of the general methods used here and from their criticisms of the specific approach. Dr. Joseph Gettrust of NRL reviewed several drafts. The funding for this work came from the Office of Naval Technology and the Advanced Environmental Acoustics Support (AEAS) program; program element 0602435N.

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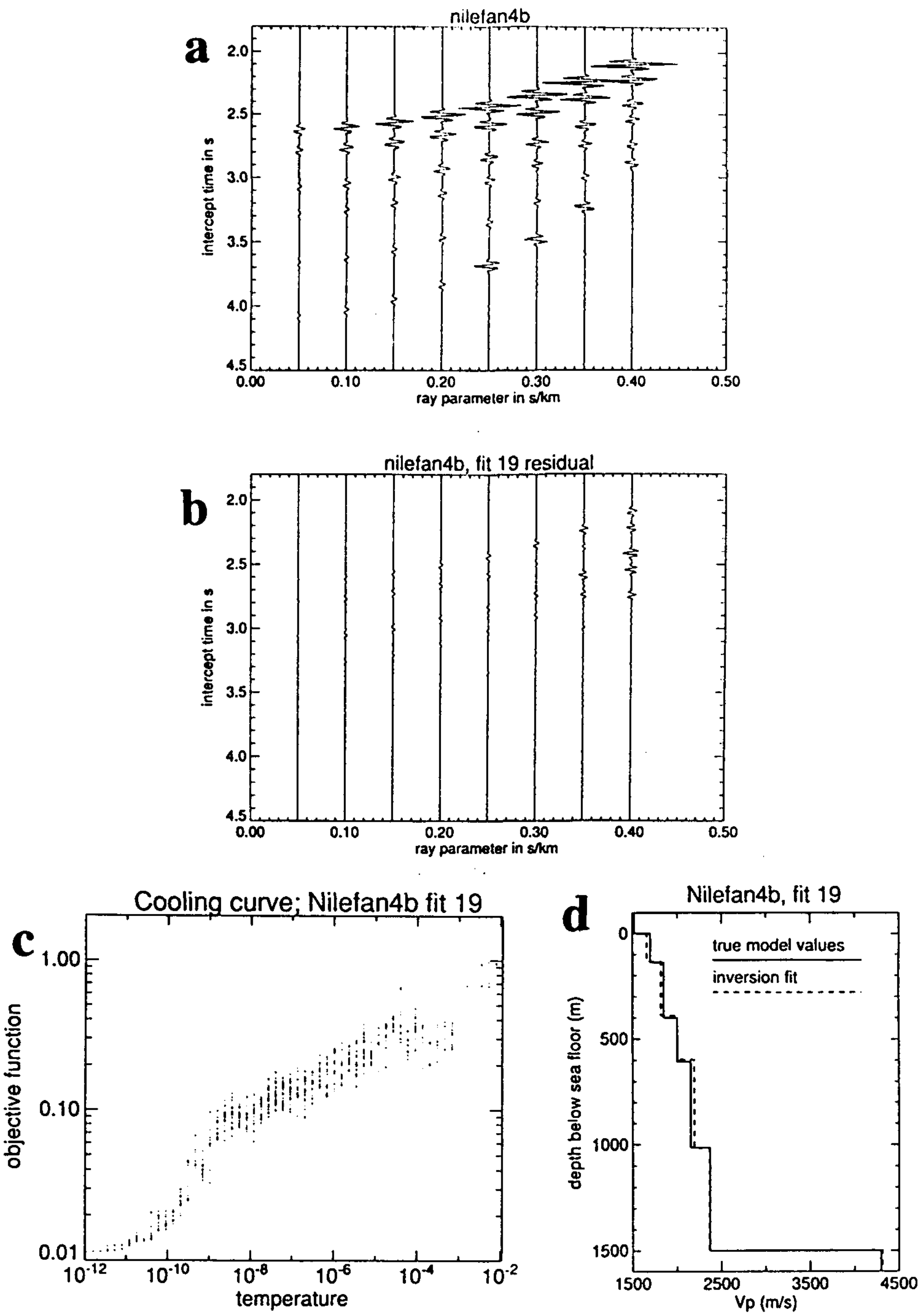


Fig. 3—a. Synthetic tau-p data from the expected environment in the Nile fan region. b. The residual of the fit for the inversion solution from the 19th run. c. The cooling curve shows the progress of the calculation from high temperatures and large objective functions on the right to low temperatures and small objective functions on the left. d compares the model used to generate the "data" with the solution found by inversion.

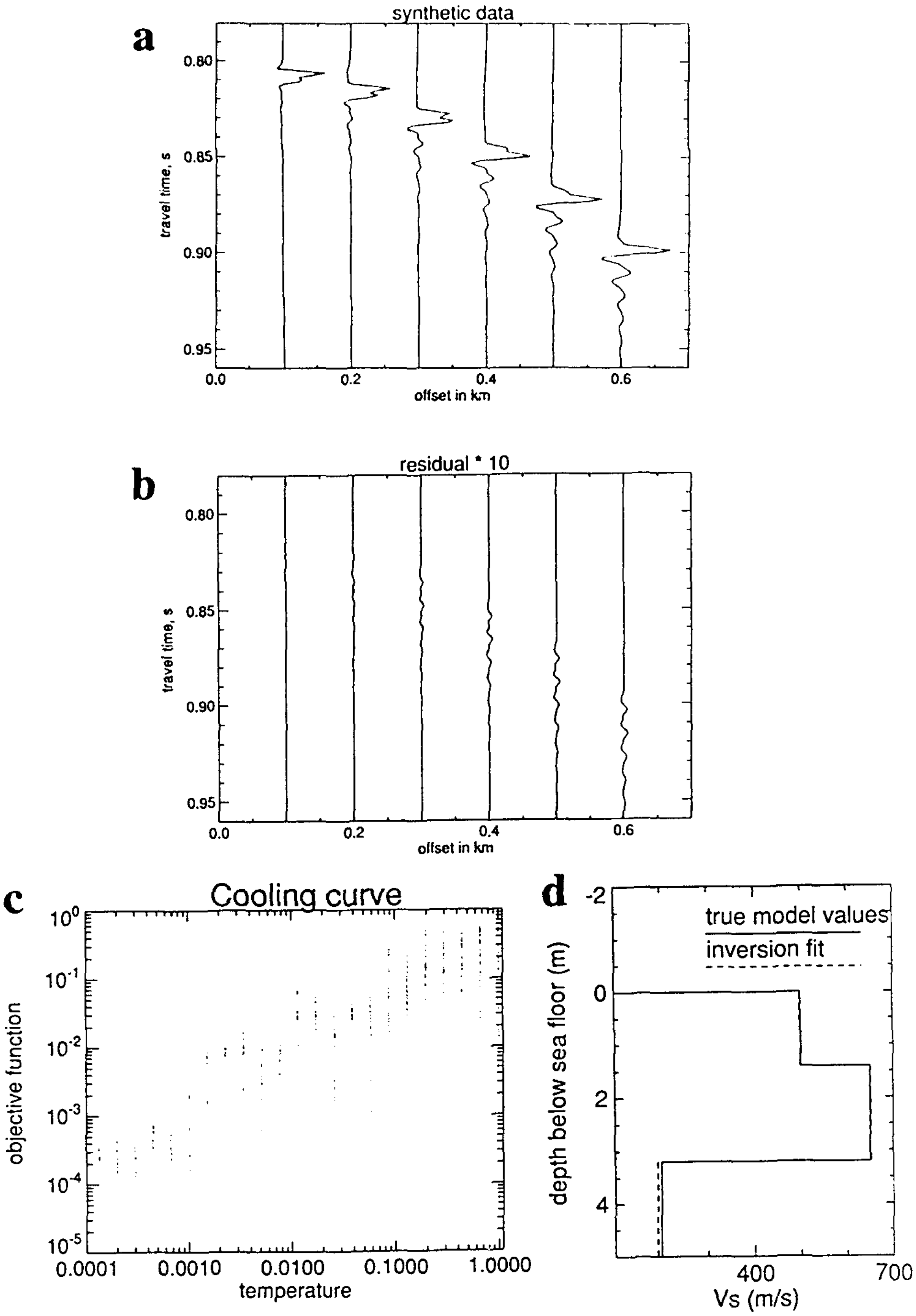


Fig. 4—a. Synthetic data in the t-x domain from a region with a high V_S at the sea floor. b. The residual of the fit for the inversion solution which solved only for V_S. c. The cooling curve shows the progress of the calculation from high temperatures and large objective functions on the right to low temperatures and small objective functions on the left. d. A comparison of the known geophysical profile and the solution from our SA inversion method.

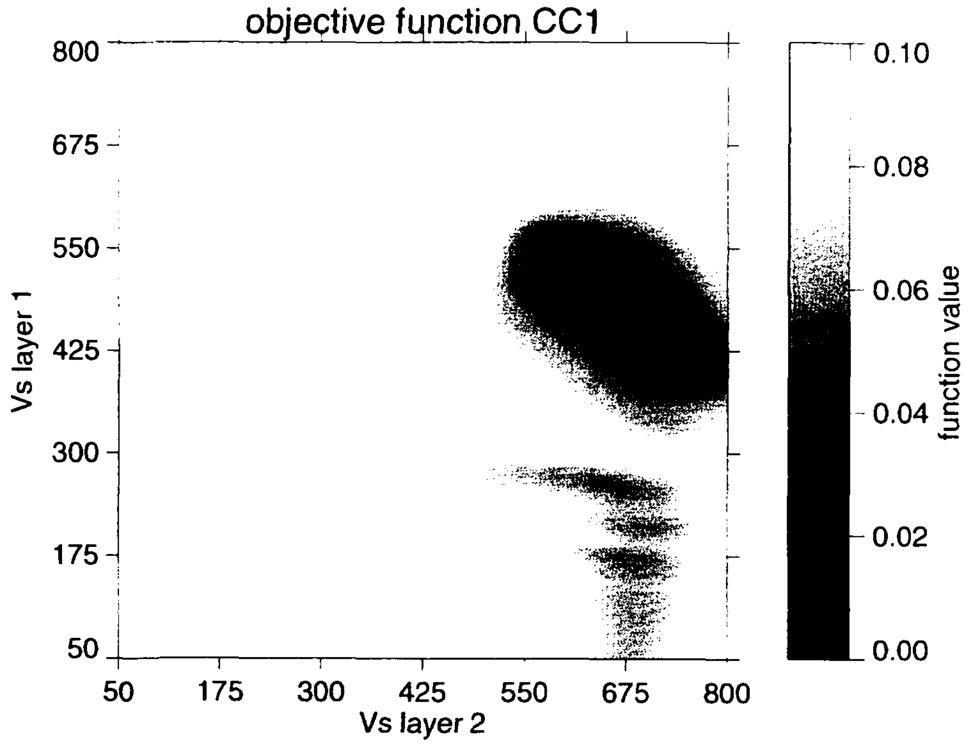
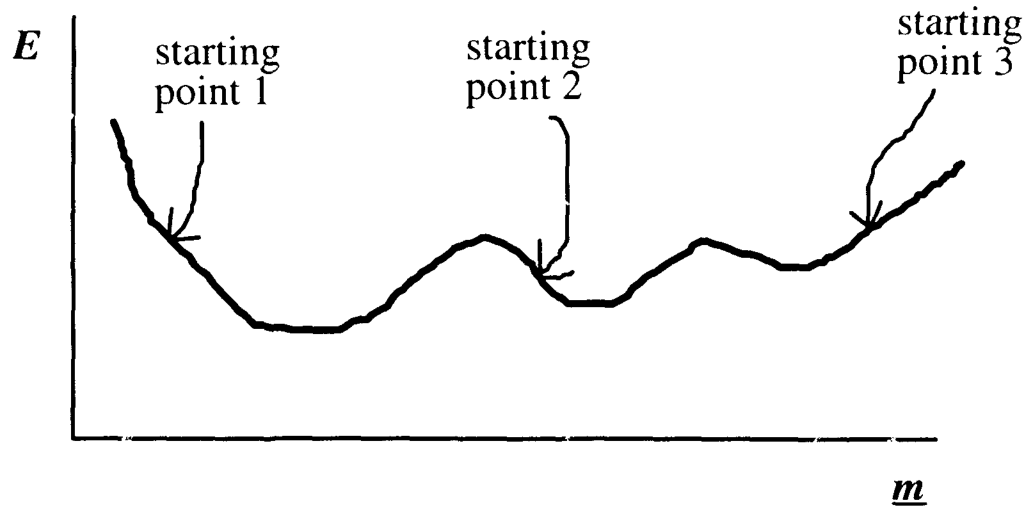


Fig. 5—A contour plot of the objective function CC1 for V_S of the top two sediment layers for the model described in Fig. 2. While the objective function varies from 1 to 0, only the range between 0.1 and 1 is shown in order that the details are brought out. The complex shape and multiple minima are a challenge for the inversion method. A linear method failed in several tries by either falling into one of the local minima or not finding a large enough gradient to refine the initial solution.

Appendix: Nonlinear Inversion and the Metropolis Algorithm

Inversion/optimization methods want to find the lowest value of $E = \|\underline{d} - \underline{G}(\underline{m})\|$ where E is the error, $\underline{d}$ is the observed data, $\underline{G}$ is the theory, and $\underline{m}$ is the earth model.



- At each step, conventional inversion/optimization goes to a lower value of E ($\underline{m}$), so the answer may not be the global minimum.
- In any step, simulated annealing (SA) may go to a higher value of E. Lower values of E are preferred, but not so strongly preferred as to send SA into a local minimum from which it will never escape.
- SA works by sampling from the Gibbs distribution:

$$P_T(\underline{m}) = \frac{e^{-E(\underline{m})/T}}{\sum_{\underline{m}} e^{-E(\underline{m})/T}}$$

while slowly lowering the temperature T without actually knowing P_T.

- **The Metropolis Algorithm**

1. Choose a random starting model $\underline{\boldsymbol{m}}$ and a high $\boldsymbol{T}$. Define $\boldsymbol{E}(\underline{\boldsymbol{m}})$ so that $|E_{\max} - E_{\min}| \cong 1$

2. Randomly perturb m_1, the first component of $\underline{\boldsymbol{m}}$, to get $\underline{\boldsymbol{m}}'$. If $E' \leq E$ then accept $\underline{\boldsymbol{m}}'$ as the new model. If $E' > E$ accept $\underline{\boldsymbol{m}}'$ with probability $e^{(E-E')/T}$

3. Visit the remaining components of $\underline{\boldsymbol{m}}$ updating in a similar manner. One visit to each component constitutes a "sweep."

4. Reduce $\boldsymbol{T}$ slightly and go to 2. Stop if $\underline{\boldsymbol{m}}$ has not changed in the last 100 sweeps.